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Temporal and spatial evolution of bursts in a fiber bundle model of creep rupture

Zsuzsa Danku[a] and Ferenc Kun[b]

Department of Theoretical Physics, University of Debrecen,

H-4010 Debrecen, P.O.Box: 5, Hungary

[a]zsuzsa.danku@phys.unideb.hu, [b]ferenc.kun@science.unideb.hu

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Abstract. We present a theoretical study of the creep rupture of heterogeneous materials based on a fiber bundle model which provides a direct connection between the microscopic fracture mechanisms and the macroscopic time evolution. In the model, material elements fail either due to immediate breaking or undergo a damage accumulating ageing process. We found that on the micro-level the competition of the two failure modes gives rise to bursts of breakings with power law distributed size and waiting time between events. We demonstrate that approaching macroscopic failure the system accelerates which can be fully described as a non-homogeneous Poissonian process for long range load sharing, however, when localization occurs breaking events get clustered. Bursts are composed of sub-avalanches which lead to a non-trivial temporal shape comparable to measurements. The pulse shape proved to be sensitive to the range of load sharing.

Introduction

Sub-critical fracture occurring under constant or periodic external loads represents an interesting scientific problem with a broad spectrum of technological applications. Creep rupture processes are often responsible for the failure of engineering constructions and they are at the core of natural catastrophes such as snow and stone avalanches, landslides, and earthquakes. To prevent catastrophic failure, monitoring techniques based on the acoustic emissions of nucleating and propagating cracks are essential. Recently, laboratory experiments and field measurements on engineering constructions have revealed that crackling events are characterized by power law distributions of burst sizes (energies) and of the waiting times between consecutive events [1]. The accelerating dynamics of rupture was found to give rise to time-to-failure power laws of the cumulative dissipated energy and of the rates of acoustic events. Here we use a fiber bundle model to investigate the statistics and time evolution of the stochastic bursting activity accompanying the process of creep rupture of heterogeneous materials. We also analyze how single acoustic bursts evolve and determine features which may be exploited for materials testing methods.

Fiber bundle Model of Damage Enhanced Creep Rupture

Our study is based on a fiber bundle model (FBM) of damage enhanced creep which was introduced recently [2,3]. A parallel bundle of fibers is considered which is organized on a square lattice. The fibers exhibit a linearly elastic behavior with the same Young modulus E and fail abruptly when the load on them σ exceeds a critical value σ_{th}. The Young modulus of fibers is fixed $E=1$, while the failure threshold σ_{th} is a random variable with a probability density function $g(\sigma_{th})$. In order to capture the aging of material elements subject to long time loading, we assume that intact fibers accumulate damage in the form of micro-cracks. The rate of damage accumulation Δc of a fiber is assumed as $\Delta c(t) = a\sigma(t)^{\gamma} dt$, where the exponent γ controls the time scale of damage accumulation, while a is a scale parameter [2]. The fibers can only tolerate a limited amount of damage and break when their total damage $c(t)$ exceeds a threshold value c_{th}, which is again a random variable with a probability density function $f(c_{th})$. Damage accumulation is the mechanism which introduces time in the model and leads to macroscopic failure of the bundle at any finite loads. Under a constant external load σ_0, after each failure event the load of the broken fiber has to be overtaken by the remaining intact ones. In the present study we considered both equal and localized load sharing. Under equal load sharing (ELS) all intact fibers get the same amount of load from the broken fiber irrespective of their distance. Hence, in this case the stress field remains homogeneous during the entire breaking process. Localized load sharing (LLS) realizes the opposite limit, i.e. we assume that the excess load is equally redistributed over the intact nearest neighbors of the failed fiber, which leads to high stress concentrations around failed regions. Our model hast two sources of disorder: the structural disorder of the material which is represented by the randomness of breaking thresholds σ^i_{th}, c^i_{th} $(i = 1, \ldots, N)$, and the heterogeneous stress field arising due to the short ranged load redistribution. Both uniform and Weibull distributions have been considered for the thresholds of immediate breaking and for ageing.

Acceleration towards failure

The separation of time scales of the slow damage process and of immediate breaking leads to a highly complex time evolution in agreement with experiments [2,3]: damaging fibers break slowly one-by-one gradually increasing the load on the remaining intact fibers. After a certain number of damage breakings the load increment becomes sufficient to induce the immediate breaking of a fiber which in turn triggers an entire burst of breakings. As a consequence, the time evolution of creep rupture occurs as a series of bursts corresponding to the nucleation and propagation of cracks, separated by silent periods of slow damaging. Figure 1*(a)* presents the time series of breaking bursts, i.e. the burst size Δ as a function of time t is plotted. Strong fluctuations of the burst size Δ can be observed, however, its average increases towards failure. The waiting times T between bursts show a similar fluctuating behavior in Fig. 1*(b)* with a general decreasing tendency towards macroscopic failure implying the acceleration of the rupture process. Macroscopic failure occurs at time t_f, which defines the lifetime of the system.

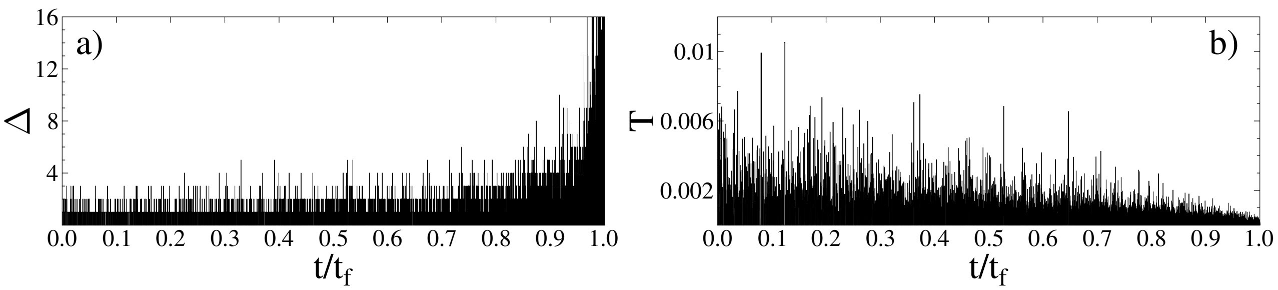

Figure 1: (a) Time series of bursts in a bundle of N=100.000 fibers, i.e. the burst size Δ is plotted as a function of time. (b) The waiting time T elapsed between consecutive events of (a). Both quantities have strong fluctuations.

In our model bursts of immediate breaking are analogous to acoustic outbreaks of experiments which can be recorded by the acoustic emission techniques. The main goal of our study is to analyze the statistics of bursts and the time evolution of the burst time series as the system approaches macroscopic failure.

To characterize the accelerating evolution of the rupture process we determined the rate of bursts n(t) as a function of the time to failure t_f-t. The equal load sharing has the consequence that the breaking of each fiber contributes to the global acceleration of the system, hence, the rate of breaking bursts increases and saturates in the vicinity of global failure.

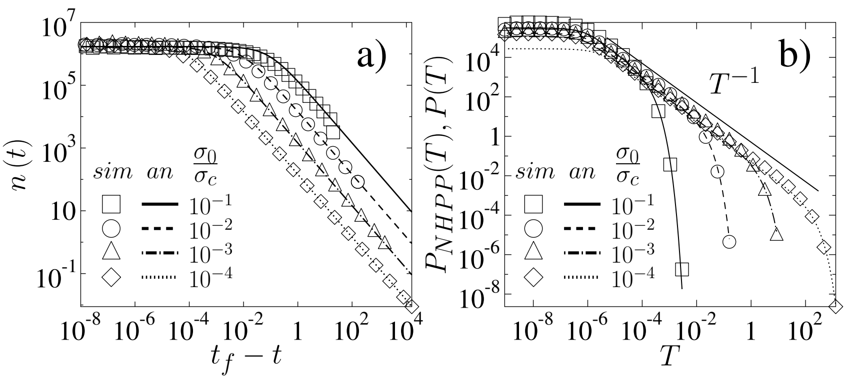

Figure 2: (a) Rate of events as a function of time to failure. The symbols are simulation results, while the continuous lines represent fits with Eq. (1). (b) Corresponding waiting time distributions (symbols). The continuous lines represent P(T) obtained from the analytic expression of non-homogeneous Poissonian processes.

It can be observed in Fig. 2*(a)* that the accelerating rate of events can be described by the Omori law well known for earthquakes [4]

$$n(t) = \frac{A}{\left[1+(t_f - t)/c\right]^p}, \tag{1}$$

where A denotes the saturation rate close to failure, c is the characteristic time scale where saturation is reached, and p is the exponent of the power law regime of $n(t)$ in Fig. 2. It is important to note that for earthquakes the Omori law Eq. 1 describes the relaxation process following major events, however, for creep rupture the accelerating crackling activity towards final failure has a similar functional form. The value of the exponent is $p = 1$ for uniformly distributed breaking thresholds, while for the Weibull distribution it proved to depend on the Weibull exponent. The temporal occurrence of bursts is controlled by the disorder of breaking thresholds in a gradually increasing stress field. It follows that the time series of events can be described as a non-homogeneous Poissonian process (NHPP) where the average of the Poissonian process is increasing according to Eq. 1 [4]. It can be observed in Fig. 2*(b)* that the probability distribution of waiting times P(T) obtained numerically can be fully described analytically by assuming NHPPs. Note that in the intermediate regime the waiting time distribution has a power law behavior $P(T) \propto T^{-z}$ with the exponent $z = 1$. When the load sharing is localized around broken fibers we found a qualitatively similar behavior, however the value of the Omori exponent p proved to depend on the external load.

Time evolution of single bursts

High precision acoustic emission measurements can capture even the evolution of single crackling events which can be inferred from the pulse shape of the signal providing valuable information also for the assessment of the damage state of construction components. Our model makes it possible to analyze the dynamics of single bursts: an avalanche typically starts with the breaking of a single fiber. As the load gets redistributed some fibers break again, creating a sub-avalanche. The avalanche evolves through sub-avalanches and stops when the remaining fibers can sustain the elevated local load. The temporal profile of bursts can be characterized by the size of sub-bursts Δ_s as a function of the internal time u with the limits $0 \le u \le W$ where the duration W of bursts is defined as the number of sub-bursts. Figure 3(a) presents the temporal profile of avalanches of the same duration W where a complex stochastic evolution is evidenced. These temporal profiles $\Delta_s(u)$ of the bursts of our model are analogous to the pulse shape of measured acoustic signals. For a quantitative characterization we calculated the average profile for fixed durations $\langle \Delta_s(u, W) \rangle$. In

Fig. 3(b) for localized load sharing the average pulses have a parabolic shape but with a right handed asymmetry. Increasing the pulse duration W the height of the curves increases but the functional form remains the same. Figure 3(c) demonstrates that rescaling $\langle \Delta_s(u, W) \rangle$ with an appropriate power α of the pulse duration W the average profiles can be collapsed on a master curve as a function of u/W. The good quality collapse implies the scaling form $\langle \Delta_s(u, W) \rangle \propto W^{\alpha} f(u / W)$, where the scaling function reads as $f(x) = Bx(1-x)^{\beta}$. The right handed asymmetry has the consequence that breaking bursts start slowly, then accelerate and stop suddenly when the crack gets pinned by some stronger material regions. When the stress field is homogeneous the pulse shape proved to be a symmetric parabola due to the dominance of structural disorder of the material during the creep process. Our results demonstrate that the temporal profile of single acoustic outbreaks depends on the range of load redistribution.

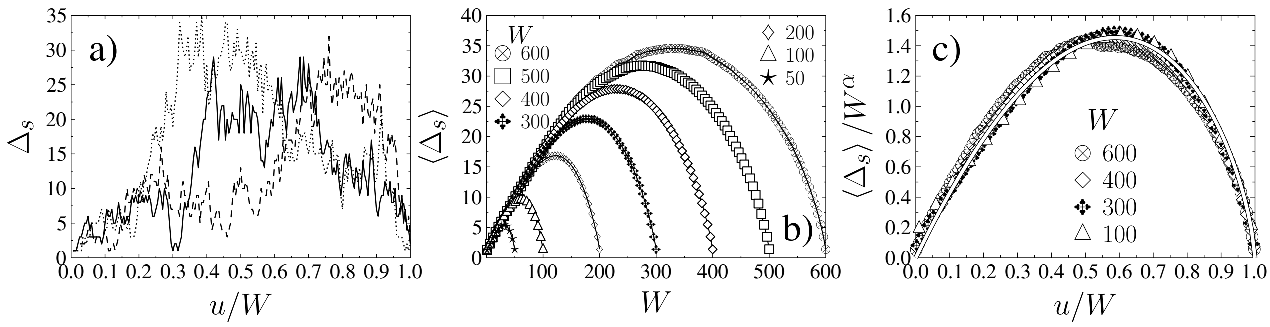

Figure 3: (a) Temporal profile of single bursts of the same duration W=200 as a function of the scaled time u/W. (b) Average pulse shape of bursts of different durations W. (c) Scaling collapse of average pulse shapes obtained with the exponent $\alpha = 0.7$. Fit parameters of the scaling function f(x) are B=4.6 and $\beta = 0.7$.

Summary

In this paper we used a fiber bundle model of damage enhanced creep of heterogeneous materials to investigate the statistics and evolution of breaking bursts during the process of creep rupture. Computer simulations revealed that the accelerating breaking activity can be described by the Omori law similar to aftershock sequences following major earthquakes and the evolution of creep can be described as a non-homogeneous Poissonian process. The model makes possible to study the dynamics of single bursts, as well. We showed that the temporal profile of bursts is analogous to the pulse shape of acoustic signals. For localized load sharing a right handed asymmetry of pulse shapes was evidenced which shows that bursts start slowly, accelerate and stop suddenly as the crack gets pinned. For homogeneous stress fields a symmetric parabolic pulse shape emerges due to the dominance of structural disorder.

Acknowledgements

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